Estimation in the primary school

Developing a key mathematical skill for life

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Paula Mildenhall discusses the importance of computational estimation in 'real life' contexts and how the language of precision and estimation can assist students to understand the usefulness of estimation.

Very recently, in the *Australian Association of Mathematics Teachers (AAMT)/Australian Industry Group quantitative report* (2014), concerns were raised that school mathematics is lacking real world application. This report highlighted the gaps between school mathematics and the requirements of the workplace. After interviewing industry representatives to find out what mathematical skills were needed in the workplace, estimation was identified as an essential skill. Particularly:

- Workers are often required to estimate approximate answers when exact calculations are not required. They need to know when to make a choice between calculation and estimation depending on a particular process. Workers are required to identify estimation requirements and select appropriate estimation methods to meet work requirements.
- Estimations can take the place of a precise calculation where precision is not required, or can be used to mentally check whether an error has been made. (p.24)

In the *Australian Curriculum* (Australian Curriculum Assessment and Reporting Authority, 2014), the general capabilities requirements also assert that estimation is important for students to live and work successfully in the 21st century.

What is estimation?

Lobato (1993, p. 350) noted a student remarking that "estimation is guessing with a little bit of problem solving". Clearly estimating involves much more than guessing. Definitions of estimation vary in the literature; Dowker (1992, p. 45) defined computational estimation (the type of estimation involved when calculating) as "making reasonable guesses as to the approximate answers to arithmetic problems, without or before actually doing the calculations". A recent and succinct definition is "finding an approximate answer to arithmetical problems without actually (or before) computing the exact answer" (Lemaire, Lecacheur, & Farioli, 2000, p. 1). There are obviously links between mental computation and computational estimation (Dowker, 2003; Hazekamp, 1986; McIntosh, 2004; Yoshikawa, 1994). Reys stated "There are two distinguishing characteristics of mental computation. It produces an exact answer, and the procedure is performed mentally, without using external devices such as pencil and paper. Mental computation is an important component of estimation in that it provides the corner-stone necessary for the diverse numeric processes used in computational estimation" (1984, p. 548). From that statement it is clear that when you estimate you undertake mental computation whereas the reverse is not true. While estimation skills are also required in measurement, spatial thinking and statistical thinking, in the remainder of this paper, the discussion will concentrate upon estimation involving computation.

How to teach estimation so that it is transferable for use in the 'real' world

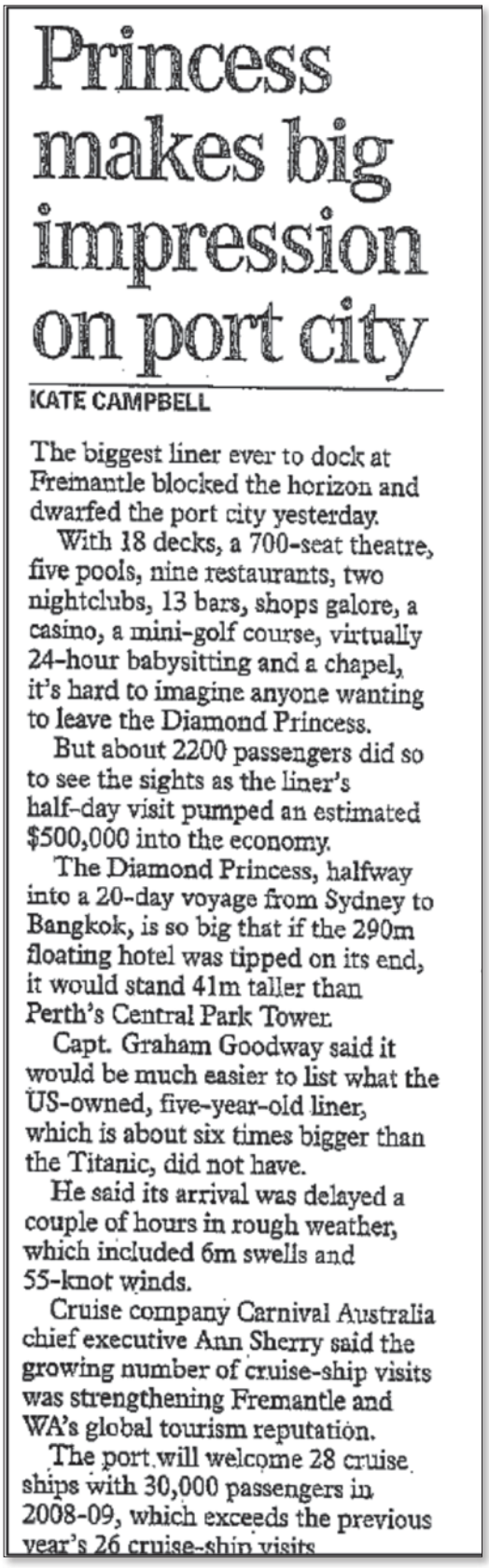
In the domain of estimation it may be productive to develop the concepts and skills in meaningful contexts (Reys & Reys, 2004). It has been asserted that "one reason is that it is easier for students to appreciate the value of estimation when it is used in the right situations; they realise that using estimation can save effort or energy" (Yoshikawa, 1994, p. 61).

Estimation in the early years

Mathematics educators have proposed that in the early years, F–2, the foundations for computational estimation can be laid. When children first enter school they can be encouraged in their conversations with teachers to use the language of estimation such as 'about' and 'nearly' when talking about numerical quantities (Leutzinger, Rathmell, & Urbatsch, 1986). In their activities children can use concrete referents such as real objects and manipulatives and focus on groups of objects in order to develop number sense and build up visual perceptions of approximate and exact amounts (Carlow, 1986). These types of activities may counter my extensive experience with estimation which suggests that the present situation is one where students are generally reluctant to estimate, preferring to calculate the actual answer and then adjust it to look like an estimate.

Estimation in the middle years

In later primary years students can explore newspapers to investigate the use of estimation in reporting events in the 'real' world. In a professional learning program conducted in Western Australia (Mildenhall, 2010) teachers trialled this activity to explore with students how estimation is used in the real world and therefore is a mathematical concept that has real world application. One of the classroom teachers 'Wendy' (a pseudonym), found an article in the day's newspaper, which was about the issue of the State's roads and it had a large amount of estimated data within the article. In a whole class discussion, the students began to identify that multiples of tens were often used when journalists provided estimates and that there were different reasons to use estimated numbers rather than provide exact ones. Wendy explained that in this lesson the students were particularly interested in the idea that estimation could be used to deceive people about the true amount! (Informal teacher interview, 27/3/2009).



Princess makes big impression on port city

KATE CAMPBELL

The biggest liner ever to dock at Fremantle blocked the horizon and dwarfed the port city yesterday.

With 18 decks, a 700-seat theatre, five pools, nine restaurants, two nightclubs, 13 bars, shops galore, a casino, a mini-golf course, virtually 24-hour babysitting and a chapel, it's hard to imagine anyone wanting to leave the Diamond Princess.

But about 2200 passengers did so to see the sights as the liner's half-day visit pumped an estimated $500,000 into the economy.

The Diamond Princess, halfway into a 20-day voyage from Sydney to Bangkok, is so big that if the 290m floating hotel was tipped on its end, it would stand 41m taller than Perth's Central Park Tower.

Capt. Graham Goodway said it would be much easier to list what the US-owned, five-year-old liner, which is about six times bigger than the Titanic, did not have.

He said its arrival was delayed a couple of hours in rough weather, which included 6m swells and 55-knot winds.

Cruise company Carnival Australia chief executive Ann Sherry said the growing number of cruise-ship visits was strengthening Fremantle and WA's global tourism reputation.

The port will welcome 28 cruise ships with 30,000 passengers in 2008-09, which exceeds the previous year's 26 cruise-ship visits

Figure 1. Newspaper article showing exact and estimation words (*The West Australian* newspaper, 4th March, 2009).

After the whole class introduction, the students worked in pairs to explore the differences between exact numbers and estimates used in a number of newspaper articles. Wendy, the teacher, scaffolded this by providing a pre-prepared table with the heading to place the numbers in the first column. In this first column, the students had written in one colour if it was an estimated number and another colour if it was an exact number. In the next column, the students had to copy the words from the article which had signalled to the students that it was estimation. The newspaper article in Figure 1 was used to explore how estimates are used.

At the end of the lesson Wendy used the Interactive White Board to summarise and record what words the students had found in the newspapers, and as she recorded the words, she asked the students whether the words suggested whether the numbers were exact or were estimates (see Figure 2). From engaging in this activity children began to identify that estimation was a useful tool used in the 'real world' to describe quantitative situations. It was highlighted how numbers do not always have to be exact and that sometimes it is satisfactory to use an approximation.

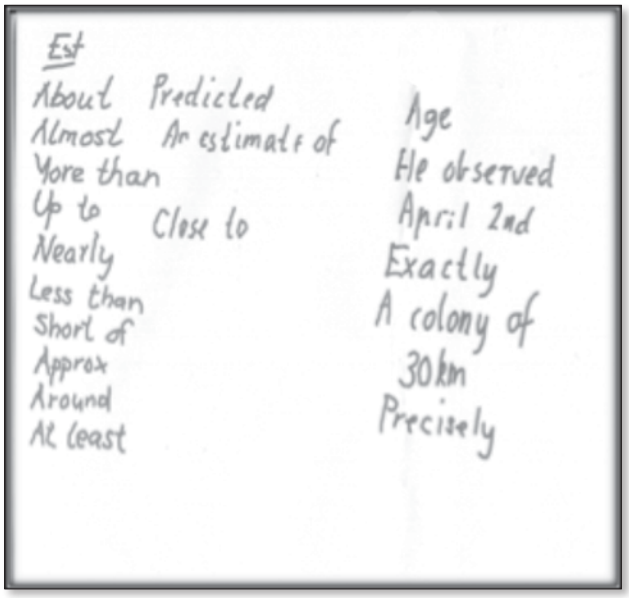


Figure 2. Words created by the children and summarised by the teacher.

Another activity, set in a surf shop, provides students with a meaningful context to develop their estimation skills (surf shops are a popular context even if schools are not close to the ocean). The game was also introduced in the professional learning program in Western Australia and trialled by the Year 6 teachers. The students enjoyed the game which encouraged them to continue using estimation strategies to solve problems. There are a variety of estimation strategies that children could use and, as research has shown that Year 6 students are able to use a number of different strategies such as bench marking, compatible numbers, and rounding (Mildenhall, 2009), this is a very worthwhile learning experience. This game could be adapted to suit different ages using different monetary amounts.

Initially the teacher introduces the game; explaining that using computational estimation strategies are very useful when shopping. Students should be encouraged to share when they have used estimation when shopping. The teacher then organises students in pairs as they will play the estimation game set up as a role play at a surf shop with a shop owner and a surfer customer. The teacher can bring in real items and model how the role play works i.e., I have a $100 note (this may be monopoly money); I want to buy a $27.67 wallet, $15.20 cap, $25.46 towel and $34.17 tee-shirt. The teacher asks "Do I have enough money?"

In pairs the students need to cut out the template pictures of items. One person plays the surf shop owner, the other the surfer. Without the surfer looking, the shopkeeper selects three items and uses a calculator to work out the exact cost of the three selected items. The partner then turns around and has 30 seconds to work out an estimate and state if $100 is enough to buy these items. If they are correct they receive a counter. It is very important at this stage to discuss how they estimated and record the exact cost and estimated cost. The pairs then reverse roles and the winner at the end is the student with the most counters. See Appendix A for the game's template.

Conclusion

For many years, mathematics educators have been asserting that computational estimation has value in the primary curriculum (Neill, 2006; Reys & Bestgen, 1981; Silver, 1994) and individual researchers have produced findings in order to support its implementation into the curriculum (Bobis, 1991; Mack, 1988). The recent collaboration between AAMT and the Australian Industry Group (2014) has reasserted the importance of

estimation and with this influential support for this aspect of mathematics it may be possible that computational estimation takes its rightful place in the Australian primary mathematics curriculum. If students learn to estimate confidently in the classroom using meaningful contexts, this confidence may be transferable for use in the real world.

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(See Appendix A next page)

Appendix A

Down at the surf shop

Work in pairs. Cut out the items in the surf shop at the bottom of the page. Then decide which one of you is the surf shop owner. If you are the surf shop owner, pick three items and using the calculator work out the total cost and add the total to the sheet (without your partner seeing). Your partner (the surfer) enters the shop and mentally estimates and decides if they could buy the three items with $100 (this should not take longer than 30 seconds). The surf shop owner tells the customer if they were right and gives them a counter if they were. The surfer then records their estimate and explains how they estimated. You swap roles and do this again. Keep on going. Do you get better at estimating?

Exact total calculated by surf shop owner	Estimate total calculated by surfer	What strategy did I use?